NAME:____________________

MATHEMATICS EXAM FOR THE ECONFIN PROGRAM
SUMMER 2010

1. **(10 points)** Compute the required matrices if

$$A = \begin{pmatrix} 0 & 8 \\ 3 & 1 \\ 5 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 10 & 1 \\ 3 & 1 & 2 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 0 \\ 7 & 5 \\ 2 & 0 \end{pmatrix}$$

a) $\mathbf{A^2}$
b) $\mathbf{A+C}$
c) $\mathbf{(A^T B)^T}$
d) $\mathbf{A + C^T}$
e) Inverse of B

2. (10 points) Solve the system Ax = d by Cramer's rule.

$$-x_1+3x_2+2x_3 = 24$$
$$x_1 + x_3 = 6$$
$$5x_2 - x_3 = 8$$

3. (10 points) Differentiate the followings.

(a) $f(x) = \ln(x^2)e^{(x^2+5)}$

(b) $f(x) = \dfrac{(3x^2+5)}{\ln(x^2-4)}$

4. (10 points) Find the critical points of the following function and specify them as relative minima/maxima or inflection point.

$$y=-x^3+4.5x^2-6x+6$$

5. (10 points) Find the total differential for the following functions:

(a) $y = (x-8)^2(7x+5)$

(b) $z = (2x+y)^2 / (x+3y)$

6. (10 points) A two product firm's revenue function is as follows:

$R = p_1Q_1 + p_2Q_2$, where Q_i and p_i represent the output level and the price of the product i, respectively.

The firm's cost function is assumed to be $C = 4Q_1^2 + 2Q_1Q_2 + 3Q_2^2$

a) Given that profit= R-C, find the profit maximizing level of output if $p_1 = 1$ and $p_2 = 2$. Verify that it is indeed maximum using the SOC test.
b) What is the maximum profit?

7. (10 points) Suppose that a person has the utility function, $U = \ln x + \ln y$ where x is the amount of hamburgers and y is the amount of soft drinks. The consumer's budget constraint is $2x+y=40$.

(a) Using the Lagrange method find the optimal amounts of consumption for both goods. Check SOC to verify the extremum points are indeed maximum.

(c) What if not allowed to eat more than 2 hamburgers?

8. (10 points) Evaluate the followings:

(a) $\int \left(5e^x - x^{-2} + \frac{3}{x} \right) dx$

(b) $\int_e^6 \left(\frac{1}{x} + \frac{14}{7x^2 + 5} \right) dx$

9. (10 points) Given the following marginal revenue function

$$dR(Q)/dQ=28Q-e^{0.3Q}$$

find the total revenue function. What initial condition can you introduce to definitize the constant of integration?

10. (10 points) Find the solution to the following and determine whether the time path is oscillatory and convergent.

$$y_{t+1}-6y_t=0, \qquad y_0=2$$